Reissner-Nordstrom Near Extremality from a Jackiw-Teitelboim Perspective

by

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ABSTRACT

In this paper, we investigate the near-extremal thermodynamics of the Reissner-Nordstrom (RN) black hole. Our methodology is based on a duality that exists between the near-horizon geometry of the near-extremal RN sector and Jackiw-Teitelboim (JT) theory. First, the described correspondence is reviewed at the classical level. Next, we consider first-order perturbations in the dual JT geometry by incorporating a quantum scalar field into the formalism. The novelty of our approach is that the matter field is endowed with a 4-dimensional pedigree. We ultimately find that back-reaction effects prohibit the JT black hole from losing all of its mass. This outcome directly implies that an RN black hole can not reach extremality and, moreover, can not even come arbitrarily close to an extremal state.

1 Introduction

A complete theory of quantum gravity currently remains a most formidable obstacle. Yet, much has been ascertained by considering gravitational effects in a semi-classical regime. At the forefront of such research has been the remarkable analogy between black hole mechanics and thermodynamic systems. Certainly, black hole analogies to the zeroth, first and second laws of thermodynamics have been well established with the appropriate identifications: the surface area of the black hole horizon with entropy [1, 2] and the surface gravity at the horizon with temperature [3]. However, a third law of black hole thermodynamics, although technically formulated [4], is still in need of much clarification before the thermodynamic analogy can be considered complete [5].

It is worth reviewing the third law as it applies to "conventional" systems. There are actually two formulations of this law; both of which can be attributed to Nernst [6]. Firstly, there is a formulation stating that the entropy of a system approaches a constant (which must be independent of all macroscopic parameters of the system) as the temperature approaches absolute zero. Typically, this constant is taken to be zero. Secondly, there is a formulation stating that it is impossible to reach absolute zero by a finite number of reversible processes. For ordinary thermodynamic systems, it is easy to show that these are equivalent statements. However, this is not the case for black holes as we shall soon see.

For sake of illustration, let us consider a Reissner-Nordstrom black hole (i.e., charged, spherically symmetric black hole in 4-dimensional spacetime) with mass M and charge Q. Examining the so-called extremal limit of $M^2 \rightarrow Q^2$, we know that the surface gravity (i.e., temperature) goes as $\kappa \propto \sqrt{M^2 - Q^2}$, while the surface area (i.e., entropy) is given by $A = 4\pi(M + \sqrt{M^2 - Q^2})^2$ (with all fundamental constants set to unity). It is immediately clear that the Hawking temperature vanishes in the extremal limit, so one would expect that the corresponding entropy would either vanish or approach a constant parameter. Alas, this is not the case, as the extremal value of entropy clearly depends on a macroscopic parameter; namely, the mass M of the black hole.

Given the ambiguous nature of the extremal-limiting case, one is forced to choose between two possible revisions of the third law as applied to black hole thermodynamics. Either that the "enthropic" formulation of the third law holds in a weakened form (i.e., the zero-temperature limit does not coincide with a vanishing entropy) or there must be a discontinuity occurring near the zero-temperature limit (i.e., zero temperature may yield a state of vanishing entropy, but such a state can not be viewed as limiting case of a finite-temperature system). This dilemma has resulted in two very different schools of thought regarding the interpretation of extremal black holes.

On one hand, an extremal black hole is often interpreted as being a well-defined (zero-temperature) limit of its non-extremal counterpart. The most compelling argument on behalf of this viewpoint has come from string-theory calculations of black hole entropy. More specifically, Strominger and others [7, 8] have considered certain classes of weakly coupled string theory for which massive string states can be represented by extremal black holes. In these works, a statistical procedure has been used to generate the Bekenstein-Hawking area law (i.e., entropy is equal to one quarter of the surface area), precisely. If such a result is valid, then clearly the extremal limit must be well-defined with entropy being a non-vanishing, mass-dependent quantity in this limit.

On the other hand, it has been conjectured that extremal black holes and non-extremal black holes are qualitatively distinct entities, with no chance of one continuously deforming into the other. In this picture, extremal black holes are indeed assigned zero entropy, assuming that they can exist at all. Support for this point of view has come from various sources. Here, we take note of studies that closely examined the following issues: topological differences between non-extremal and extremal geometries [9, 10, 11, 12], extremal breakdowns occurring at perturbative order [13, 14, 15], non-thermal behavior of an "incipient" extremal black hole [16] and violations in the second law for finite-entropy extremal black holes [17].

Suffice it to say, given these two conflicting viewpoints, there is still no consensus on the subject of extremal thermodynamics.

Let us now consider a recent paper, of relevance to this subject, by Fabbri, Navarro and Navarro-Salas [18]. These authors demonstrated a direct correspondence between the near-horizon behavior of near-extremal Reissner-Nordstrom (RN) black holes and a static geometry in AdS_2 (i.e., 2-dimensional anti-de Sitter spacetime).[1] What is particularly interesting

[1]This dual behavior originates, quite naturally, by way of the AdS_2/CFT_1 correspondence [19, 20, 21, 22].

about this correspondence is that it implies a duality between near-extremal RN black holes and near-massless black holes in Jackiw-Teitelboim (JT) theory [23]. Such a duality has intriguing ramifications regarding the third law of thermodynamics for the following reason. Massless JT black holes have both a vanishing temperature and a vanishing entropy. That is, with regard to JT theory, the third law of thermodynamics holds naturally in both of Nernst's formulations. Given the eternal vagueness of what constitutes the "true physical picture" of any given system, this duality should be taken seriously as supporting the viewpoint of a well-defined extremal limit.

Fabbri et al. [18] continued on in their paper with an analysis of back-reaction effects due to the presence of minimally coupled scalar fields. On this perturbative level, their rigorous analysis supported a well-defined extremal limit. However, here, we have to take issue with one point. The matter fields were assumed to be minimally coupled with regard to the JT black hole theory. Since the JT action, in this context, has its origins in a 4-dimensional theory, we feel that any matter fields should be subjected to the same criteria. That is, it can be argued that the matter fields should be, for instance, minimally coupled in the original 4-dimensional theory. In this case, the form of coupling in the dual 2-dimensional theory can only be ascertained after implementing the same dimensional-reduction ansatz as applied to the classical action. An investigation into this very matter is the topic of the current paper.

The content of this paper is organized as follows. In Section 2, we review the correspondence that was found between near-extremal RN black holes and near-massless JT black holes [18]. Then, in Section 3, we consider quantum back-reaction effects to first-perturbative order. (The philosophy that underlies our approach is based on a study by Anderson, Hiscock and Taylor [14].) In particular, the quantum-corrected form of the surface gravity (or, equivalently, temperature) is calculated. After which, we determine if a massless black hole complies with a non-negative surface gravity; a necessary condition for any black hole thermodynamic system to have physical meaning. With an eye to completeness, this analysis is carried out for both a matter field that is minimally coupled in the effective 2-dimensional theory and in the original 4-dimensional theory. Note that, for the sake of brevity, much of the technical details of this section have been left to previously published papers by Kunstatter and this author [15, 24]. Finally, Section 4 closes with a summary and discussion of the results.

2 Near-Extremal Reissner-Nordstrom Black Holes

Fabbri, Navarro and Navarro-Salas [18] have recently demonstrated an explicit correspondence between near-extremal Reissner-Nordstrom (RN) black holes and near-massless black holes of 2-dimensional AdS gravity. In this section, we review this duality, as it is essential to the later analysis of this paper.

Let us begin with the 4-dimensional Einstein-Maxwell action:[2]

$$I^{(4)} = \frac{1}{16\pi G} \int d^4x \sqrt{-g^{(4)}} \left[R^{(4)} - F^{AB} F_{AB} \right], \tag{1}$$

where G is the 4-dimensional Newton constant and F_{AB} is the Abelian field-strength tensor $(A, B = 0, 1, 2, 3)$.

The unique static and spherically symmetric solution of this action can be described by the well-known RN metric:

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{2}$$

where M and Q represent the conserved quantities of black hole mass and black hole charge (respectively). If $M^2G^2 > GQ^2$, this is the solution for a charged, non-extremal black hole with two distinct horizons. These are given by:

$$r_\pm = GM \pm \sqrt{G^2M^2 - GQ^2}. \tag{3}$$

The thermodynamic properties of non-extremal, highly symmetric black holes are well established [1, 2, 3]. In the RN case, the associated entropy and temperature are respectively found to be:

$$S_{BH} = \frac{A_+}{4\hbar G} = \frac{\pi r_+^2}{\hbar G}, \tag{4}$$

$$T_H = \frac{\hbar \kappa_+}{2\pi} = \hbar \frac{r_+ - r_-}{4\pi r_+^2}, \tag{5}$$

[2]For the duration of the paper, we work in units such that the speed of light and Boltzmann's constant are set equal to unity.

where A_+ is the surface area and κ_+ is the surface gravity with respect to the outermost horizon.

For the case of extremal black holes (i.e., when $G^2M^2 = GQ^2$ or $r_- = r_+$), the associated thermodynamic properties remain an open issue (as discussed in Section 1). However, we can still safely consider a "near-extremal regime" by setting $\Delta M = M - M_o$, where $M_o^2 = Q^2/G$ (with the charge Q assumed to be a fixed quantity). Then to leading order in $\sqrt{\Delta M}$:

$$r_+ = GM_o + G\sqrt{2M_o\Delta M}, \tag{6}$$

$$\Delta S_{BH} \equiv S_{BH}(M,Q) - S_{BH}(M_o) = \frac{4\pi GM_o}{\hbar}\sqrt{\frac{M_o\Delta M}{2}} \tag{7}$$

$$\Delta T_H \equiv T_H(M,Q) - T_H(M_o) = \frac{\hbar}{2\pi GM_o^2}\sqrt{2M_o\Delta M}, \tag{8}$$

where $S_BH(M_o) = \pi GM_o^2/\hbar$ and $T_H(M_o) = 0$.

It is well known that the imposition of spherical symmetry on the 4-dimensional Einstein-Maxwell action leads to a 2-dimensional effective theory. After this procedure, the dimensionally reduced action can be expressed as follows (see, for example, refs.[25, 26]):

$$I = \int d^2x\sqrt{-g}\left[\frac{\phi^2}{4l^2}R + \frac{1}{2l^2}(\nabla\phi)^2 + \frac{1}{2l^2} - \frac{Q^2}{2\phi^2}\right], \tag{9}$$

where the "dilaton" ϕ is identifiable with the radius of the symmetric two-sphere, $l^2 = G$, the conserved charge is (still) given by Q, and all geometric quantities have been defined with respect to the resultant 1+1-dimensional manifold.

It is convenient to eliminate the kinetic term in Eq.(9) by way of the following field reparametrization [27]:

$$\overline{\phi} = \frac{\phi^2}{4l^2}, \tag{10}$$

$$\overline{g}_{\mu\nu} = \sqrt{\overline{\phi}}g_{\mu\nu}. \tag{11}$$

The reparametrized action then takes on the following form:

$$I = \int d^2x\sqrt{-\overline{g}}\left[\overline{\phi}R(\overline{g}) + \frac{1}{l^2}\overline{V}_Q(\overline{\phi})\right], \tag{12}$$

where:

$$\overline{V}_Q(\overline{\phi}) = \frac{1}{2\sqrt{\overline{\phi}}}\left[1 - \frac{Q^2}{4\overline{\phi}}\right]. \tag{13}$$

It is pertinent to this analysis that the extremal configuration (i.e., the extremal RN limit assuming its existence) can be recovered when the "potential" $\overline{V}_Q(\overline{\phi})$ vanishes. That is, when:

$$\overline{\phi} = \overline{\phi}_o \equiv \frac{Q^2}{4}. \tag{14}$$

With this in mind, let us now define $\tilde{\phi} = \overline{\phi} - \overline{\phi}_o$ and expand the action (12) about the extremal configuration. To first order in $\tilde{\phi}$, the following is obtained:

$$I = \int d^2x\sqrt{-\overline{g}}\left[\tilde{\phi}R(\overline{g}) + \frac{1}{l^2}\tilde{V}_Q(\tilde{\phi})\right], \tag{15}$$

where:

$$\tilde{V}_Q(\tilde{\phi}) = \left.\frac{d\overline{V}_Q}{d\overline{\phi}}\right|_{\overline{\phi}_o}\tilde{\phi} = \frac{4}{|Q|^3}\tilde{\phi}. \tag{16}$$

Henceforth, we drop the tildes and bars; thus considering the following action:

$$I = \int d^2x\sqrt{-g}\phi\left[R(g) + 2\frac{\lambda}{l^2}\right], \tag{17}$$

where $\lambda = 2/|Q|^3$. This is simply the action for 2-dimensional AdS gravity for which black hole solutions are known and commonly referred to as Jackiw-Teitelboim (JT) black holes [23].

It can be readily shown that, for a static gauge, the general solution of the JT action (17) is expressible as follows:

$$ds^2 = -(\lambda\frac{x^2}{l^2} - ml)dt^2 + (\lambda\frac{x^2}{l^2} - ml)^{-1}dx^2, \tag{18}$$

$$\phi = \frac{x}{l}, \tag{19}$$

where m represents the conserved mass of the JT black hole. Moreover, with straightforward application of Ref.[28] (applicable to a generic 2-dimensional dilaton theory), we are able to identify the following thermodynamic properties:

$$S_{JT} = \frac{4\pi}{\hbar}\phi_+, \tag{20}$$

$$T_{JT} = \frac{\hbar\lambda}{2\pi l}\phi_+, \tag{21}$$

where $\phi_+ = x_+/l = \sqrt{lm/\lambda}$ is the horizon value of the dilaton field.

Recalling that $\lambda = 2/|Q|^3$, $l^2 = G$ and $Q^2 = GM_o^2$, and also identifying m with ΔM, we can easily show the following:

$$S_{JT} = \frac{4\pi G M_o}{\hbar}\sqrt{\frac{M_o \Delta M}{2}}, \tag{22}$$

$$T_{JT} = \frac{\hbar}{2\pi G M_o^2}\sqrt{2M_o\Delta M}. \tag{23}$$

A direct comparison of these outcomes with Eqs.(7,8) yields a couple of intriguing identifications:

$$S_{JT} = \Delta S_{BH}, \tag{24}$$

$$T_{JT} = \Delta T_H. \tag{25}$$

That is, the thermodynamic properties of a near-extremal black hole coincide with JT thermodynamics.

The massless ($m \to 0$) limit in the JT sector appears to be a well-defined limiting procedure. Moreover, in this massless limit, $S_{JT} \to 0$ as $T_{JT} \to 0$; so the enthropic formulation of the third law of thermodynamics has been explicitly realized. With these observations, as well as the observed duality, it is tempting to conclude that extremal RN black holes can be interpreted as a limiting case of non-extremal solutions. However, such a conclusion is premature until back-reaction effects have properly been accounted for.

3 Perturbed Jackiw-Teitelboim Black Holes

3.1 General Setup

Generally speaking, the emission of Hawking radiation is expected to have repercussions on the underlying black hole geometry. This effect should be of particular importance near the extremal (or, for JT theory, massless) limit, where even the smallest changes in black hole mass can significantly deform the background geometry. To investigate the implications of such back-reaction effects, we will suitably adapt an approach suggested by Anderson et al. [14].

Let us begin here by considering a JT black hole in a state of thermal equilibrium with a quantized matter field. This equilibrium state ensures that the perturbed geometry continues to be static and, hence, can be generically expressed in the following manner:

$$ds^2 = -e^{2\omega(x)}\left[\lambda\frac{x^2}{l^2} - ml - l\mu(x)\right]dt^2 + \left[\lambda\frac{x^2}{l^2} - ml - l\mu(x)\right]^{-1}dx^2. \quad (26)$$

With this ansatz, the quantum corrections are expressed in terms of a "mass correction" $\mu(x)$ and another function $\omega(x)$; both of which are required to vanish in the classical ($\hbar \to 0$) limit. For future convenience, we will introduce a perturbative parameter $\epsilon \sim \hbar << 1$ and write:

$$\mu(x) = \epsilon\mu_1(x) + \mathcal{O}(\epsilon^2), \quad (27)$$

$$e^{2\omega(x)} = 1 + 2\epsilon\omega_1(x) + \mathcal{O}(\epsilon^2). \quad (28)$$

Let us next consider the semi-classical Einstein equation:

$$G^\mu_\nu =< T^\mu_\nu > . \quad (29)$$

Using the quantum-corrected metric (26) to describe the Einstein tensor (G^μ_ν), one finds the following to first order in ϵ (see, for example, Ref.[29]):

$$\epsilon\mu_1' = - < T^t_t >, \quad (30)$$

$$\epsilon\omega_1' = \frac{l^3}{2(\lambda x^2 - l^3 m)}\left[< T^x_x > - < T^t_t >\right], \quad (31)$$

where primes indicate differentiation with respect to x.

To check the consistency of the perturbed solution when near extremality, it is useful to consider the quantum-corrected surface gravity. The premise being that black hole thermodynamics can only have physical meaning when the surface gravity (or, equivalently, the temperature) maintains a non-negative value. Using standard calculational procedures [30], we have:

$$\kappa = \frac{e^{-\omega}}{2}\left[\lambda\frac{x^2}{l^2} - lm - l\mu\right]'\Bigg|_{x=x_+ + q}, \quad (32)$$

where $x_+ = \sqrt{l^3 m/\lambda}$ is the classical horizon and q is its quantum deformation. Up to first order in ϵ, this expression can be written:

$$\kappa = [1 - \epsilon\omega_1(x_+)] \frac{\lambda}{l^2} x_+ + \epsilon \frac{\lambda}{l^2} q_1 - \epsilon \frac{l}{2} \mu_1'(x_+), \tag{33}$$

where $q = \epsilon q_1 + \mathcal{O}(\epsilon^2)$.

In principle, μ_1 and ω_1 can be directly obtained from Eqs.(30,31), while the one-loop horizon shift can be expressed as:[3]

$$q_1 = \frac{l^3}{2\lambda x_+} \mu_1(x_+). \tag{34}$$

3.2 Minimal Coupling in 2-D

If we are to proceed with an explicit calculation, it is necessary to make some assumptions regarding the nature of the quantized matter field. Let us begin with the simple choice of a massless scalar field (f) that is minimally coupled in the 2-dimensional theory. The revised (total) action thus becomes:

$$I_{TOT} = I_{JT} - \frac{\hbar}{2} \int d^2x \sqrt{-g} (\nabla f)^2, \tag{35}$$

where I_{JT} is the action of Eq.(17). After integrating out the matter field and then taking the vacuum limit, one is known to obtain a quantum effective action of the following form [31]:

$$I_{TOT} = I_{JT} - \frac{\hbar}{96\pi} \int d^2x R \frac{1}{\Box} R. \tag{36}$$

At this point, we defer the details of the stress-tensor calculation (and related quantities) to Ref.[24]. In this prior study, first-order quantum corrections to the JT model were explicitly calculated for the case of minimally coupled scalar fields. The following results, which are of pertinence to the current study, were found:[4]

$$\epsilon \mu_1'(x_+) = \frac{\hbar\lambda}{24\pi l^2}, \tag{37}$$

[3] The relation follows from a Taylor expansion of the defining relation: $l^{-2}\lambda(x_+ + q)^2 - lm - l\mu = 0$.

[4] There is a minor modification to the prior results due to the extra factor of λ in the cosmological-constant term. This additional consideration only trivially affects the calculations.

$$\epsilon\omega_1(x_+) = 0 + \mathcal{O}\left(\frac{1}{L}\right), \tag{38}$$

$$\epsilon q_1 = \frac{\hbar l}{48\pi}. \tag{39}$$

Here, L is the extent of the system's outer boundary, which (for our purposes) can be taken to infinity.[5] [6] Substituting these results into Eq.(33) for the first-order surface gravity, we have:

$$\kappa = \frac{\lambda}{l^2}x_+ = \sqrt{\frac{\lambda m}{l}}. \tag{40}$$

To review, we have considered the case of a minimally coupled scalar field; that is, minimally coupled with regard to the 2-dimensional JT theory. We find that the surface gravity does indeed remain non-negative, even in the massless limit, when first-order perturbative effects are considered. However, as priorly discussed, this is not necessarily an appropriate form for the coupling. In Section 1, we have argued that the matter fields should have some sort of 4-dimensional pedigree. One simple, sensible choice is the condition of a minimally coupled scalar in the 4-dimensional model. In this case, the coupling in the 2-dimensional theory would be fixed by the dimensional-reduction process and would, presumably, no longer be minimal. We consider this scenario next.

3.3 Minimal Coupling in 4-D

Let us now consider a massless scalar field (f) that is minimally coupled in the 4-dimensional theory. The revised (total) action for the four dimensional theory can then be written:

$$I^{(4)}_{TOT} = I^{(4)} - \frac{\hbar}{16\pi G}\int d^4x\sqrt{-g^{(4)}}(\nabla^{(4)}f)^2, \tag{41}$$

[5]Since we are considering a black hole in thermal equilibrium, it is necessary to enclose the system in a "box" [32].

[6]One might anticipate a similar box contribution arising in the calculation of $\mu_1(x_+)$ and (hence) q_1. However, in this case, we have assumed the constant of integration to be absorbed into the "renormalized" zeroth-order mass (m).

where $I^{(4)}$ is the Einstein-Maxwell action of Eq.(1). Again imposing spherical symmetry, we obtain a dimensionally reduced form:

$$I_{TOT} = I - \frac{\hbar}{2l^2} \int d^2x \sqrt{-g} \phi^2 (\nabla f)^2, \tag{42}$$

where I is the reduced action of Eq.(9).

Following the same pattern of field reparametrization and expansion as in Section 2, we eventually find:

$$I_{TOT} = I_{JT} - 2\hbar \int d^2x \sqrt{-\overline{g}} \tilde{\phi} (\overline{\nabla} f)^2, \tag{43}$$

where I_{JT} is the JT action of Eq.(15) (or Eq.(17)) and we have explicitly shown the tilde and bar notation for the sake of clarity. Notably, the dilaton-matter coupling is precisely that obtained in the dimensional reduction (from three to two dimensions) of a BTZ black hole, assuming minimal coupling in the higher-dimensional theory [33, 34]. Hence, we can treat this model as a dimensionally reduced (non-rotating) BTZ black hole.

After integrating out the matter field and then taking the vacuum limit, one can exploit the 2-dimensional trace conformal anomaly [35, 36, 37] to obtain the following action [38]:

$$I_{TOT} = I_{JT} - \frac{\hbar}{96\pi} \int d^2x \sqrt{-g} \left[R \frac{1}{\Box} R - \frac{3}{\phi^2} (\nabla\phi)^2 \left(\frac{1}{\Box} R - \ln \eta^2 \right) - 6 \ln(\phi) R \right], \tag{44}$$

with tildes and bars again being suppressed. Note that η is an arbitrary parameter that arises out of the renormalization procedure [39].

A brief aside. It would be remiss of us not to point out that such a reduction process has recently been criticized. In particular, it has been shown that the procedures of quantization and reduction do not necessarily commute [40]. Also, the reduced form of the action may be missing non-local terms that originate from the conformally invariant part of the action [41]. However, it remains uncertain as to what (if any) effect these considerations would have on the qualitative features of a first-order, semi-classical analysis.

Now let us return our attentions to the non-local action of Eq.(44). At this point, one can proceed by first re-expressing this action in an equivalent localized form. (See, for instance, Refs.[42, 24].) This should be followed by the careful imposition of boundary conditions (which must be suitable for a black hole in thermal equilibrium). After which, the first-order stress tensor can be evaluated in a straightforward manner. We again defer to Refs.[15, 24] for the intricate details of this calculation (it would also be useful to consult Refs.[13, 29, 42, 43]) and simply quote the pertinent results:

$$\epsilon\mu_1'(x_+) = \frac{\hbar\lambda}{6\pi l^2}, \tag{45}$$

$$\epsilon\omega_1(x_+) = -\frac{\hbar}{32\pi}\sqrt{\frac{\lambda}{lm}}\ln(4lm) + \mathcal{O}\left(\frac{1}{L}\right), \tag{46}$$

$$\epsilon q_1 = \frac{\hbar l}{96\pi}\left[3\ln(4lm) - 1\right]. \tag{47}$$

Note that the arbitrary (renormalization) constant η has been suitably fixed to eliminate it from the field equations.

Substituting the above results into Eq.(33), we obtain the following revision to the first-order surface gravity:

$$\kappa = \sqrt{\frac{\lambda m}{l}} + \frac{\hbar\lambda}{16\pi l}\ln(4lm) - \frac{3}{32}\frac{\hbar\lambda}{\pi l}. \tag{48}$$

We now recall the criteria that κ be non-negative for a legitimate black hole solution. Thus, from the above result, it is immediately apparent that some sort of lower bound must be imposed on the zeroth-order (but renormalized) mass m. (Notice that $\kappa \to -\infty$ as $m \to 0$.) For illustrative purposes, let us assume that the lower bound on mass (m_b) occurs when $4lm \sim 1$. It thus follows that:

$$\begin{aligned} \sqrt{m_b} &\approx \frac{3\hbar}{32\pi}\sqrt{\frac{\lambda}{l}} \\ &\approx \frac{3\hbar}{32\pi G M_o^2}\sqrt{2M_o}, \end{aligned} \tag{49}$$

where the lower line has been expressed in terms of the 4-dimensional parameters of M_o (the mass of the black hole at theoretical extremality) and G (the Newton gravitational constant).

Let us further recall that the JT black hole mass (m) has been identified with ΔM; that is, the mass deviation from extremality in the RN black hole. Hence, the lower bound on m directly implies that the RN black hole can

not reach an extremal state. Furthermore, we can apply our quantitative result for m_b to determine lower bounds on the RN entropy and temperature (cf. Eqs.(7,8)):

$$S_{BH}(M,Q) \geq S_{BH}(M_o) + \frac{3}{8}, \tag{50}$$

$$T_H(M,Q) \geq \frac{3}{32\pi^2}\frac{\hbar^2}{G^2 M_o^3}. \tag{51}$$

The situation is, of course, more complex than this on account of the neglected logarithmic term in Eq.(48). (Not to mention the considerations of Refs.[40, 41].) However, the qualitative implications of this analysis remain quite clear. A non-extremal black hole can **not** continuously evolve to a state of extremality. Rather, the quantum back reaction will hinder the evaporation process; with the black hole ultimately "freezing" at some non-zero temperature that is related to the Planck scale. One might go so far as to say that back-reaction effects intrinsically protect the third law of thermodynamics.

4 Conclusion

In the previous paper, we have considered near-extremal Reissner-Nordstrom black holes. To begin the analysis, we reviewed a duality [18] that exists between the near-extremal RN sector and Jackiw-Teitelboim theory [23]. This correspondence was established by a process of dimensional reduction, followed by an appropriate conformal transformation of the metric (along with a redefinition of the dilaton field that arises out of the reduction ansatz). With a series of identifications, it was demonstrated that the thermodynamic properties of the JT black hole are indeed equivalent to deviations (from extremality) in the original RN theory. Also of note, the massless limit in the JT solution effectively describes the extremal limit of the RN black hole.

At a first glance, the massless JT black hole appears to be at the terminus of a well-defined limiting procedure. This is certainly a valid assessment from a classical viewpoint; however, we felt it was necessary to clarify the situation with regard to quantum-perturbative effects. In particular, we incorporated a (massless) quantum scalar field and then considered first-order deformations to the JT geometry. With the assumption of a minimally coupled matter field in two dimensions, we found that the well-defined massless limit remains intact. However, the underlying assumption is decidedly incompatible with the 4-dimensional origins of the theory. With this in mind, we also examined the case of a minimal coupling in 4-dimensions; thus giving the matter field a 4-dimensional pedigree.

To begin the revised first-order analysis, we considered Einstein-Maxwell theory minimally coupled to a quantum scalar field. After repeating the procedures of dimensional reduction and field reparametrization, we found that the total action effectively mimics that of a dimensionally reduced (non-rotating) BTZ model [33, 34]. With this realization, we were able to directly apply the results of a prior study [15, 24] and calculate the first-order corrections to the JT geometry.

Guided by a related paper [14], we invoked a condition of non-negative surface gravity (or, equivalently, temperature) as an appropriate litmus test for the validity of a low-mass JT solution. In this manner, we were able to establish a finite lower bound on the JT black hole mass. For values of mass falling below this bound, first-order perturbative effects drive the surface gravity below zero, leading to physically unacceptable solutions. Given the RN-JT duality (existing in the near-extremal RN sector), the lower bound on JT mass directly implies a finite lower bound on the temperature of a RN black hole. That is, because of back-reaction effects, the RN black hole will be prohibited from attaining an extremal state. Moreover, The RN black hole can **not** even come arbitrarily close to an extremal solution; rather, it will "freeze" at a finite temperature that is related to the Planck scale.

Given the distinct topological differences that exist between the extremal and non-extremal sectors [9, 10, 11, 12], as well as the third law of thermodynamics, it is not surprising that a discontinuity exists between the two solutions. What may be more of a revelation is the finite "separation"; that is, a non-extremal black hole can not come arbitrarily close to extremality. In defense of this notion, we point out a recent investigation into the physical spectra of charged black holes [44]. In this study, it was generically shown that extremal black holes can **not** be achieved (at the quantum level) due to vacuum fluctuations in the horizon. With the presumption that such fluctuations are of Planck-scale order, this result coincides nicely with our findings.

In spite of our conclusions and similar ones elsewhere in the literature, there remain many open questions with regard to the status of extremal black holes. For instance, can they exist, and (if so) can they be understood as thermodynamic systems. The definitive answers may have to await a comprehensive theory of quantum gravity.

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